

Evaluate $\int x \sec^2 x \, dx$.

$$= \underbrace{x \tan x}_{(2)} + \underbrace{\ln |\cos x|}_{(3)} + C$$

SCORE: ____ / 5 PTS

u dv

x $\sec^2 x$

1 $\tan x$

0 $-\ln |\cos x|$

Evaluate $\int e^{-3x} \sin 4x \, dx$. $= \underbrace{-\frac{1}{3} e^{-3x} \sin 4x}_{\text{u}} - \underbrace{\frac{4}{9} e^{-3x} \cos 4x}_{\text{dv}}$

SCORE: ____ / 5 PTS

$\frac{u}{\sin 4x}$ $\frac{dv}{e^{-3x}}$

$4 \cos 4x$ $-\frac{1}{3} e^{-3x}$

$16 \sin 4x$ $\frac{1}{9} e^{-3x}$

$-\frac{16}{9} \int e^{-3x} \sin 4x \, dx$ (1) EACH

$\frac{25}{9} \int e^{-3x} \sin 4x \, dx = -\frac{1}{3} e^{-3x} \sin 4x - \frac{4}{9} e^{-3x} \cos 4x$

$\int e^{-3x} \sin 4x \, dx = \underbrace{-\frac{3}{25} e^{-3x} \sin 4x - \frac{4}{25} e^{-3x} \cos 4x + C}_{\text{final answer}}$

Evaluate $\int \sin^5 x \cos^4 x \, dx$.

$u = \cos x$ ①
 $du = -\sin x \, dx$

SCORE: ____ / 5 PTS

$$\int -\sin^4 x \cos^4 x \cdot -\sin x \, dx$$

$$= -\int (1-u^2)^2 u^4 \, du \quad \text{①}$$

$$= \int (-u^4 + 2u^6 - u^8) \, du$$

$$= -\frac{1}{5}u^5 + \frac{2}{7}u^7 - \frac{1}{9}u^9 + C = -\frac{1}{5}\cos^5 x + \frac{2}{7}\cos^7 x - \frac{1}{9}\cos^9 x + C \quad \text{①}$$

Prove the reduction formula $\int \sin^n u \, du = -\frac{1}{n} \sin^{n-2} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du$.

SCORE: ____ / 7 PTS

NOTE: You must show how to get this formula.

You will receive 0 credit if your "proof" is differentiating both sides of the equation.

$$\begin{array}{cc} \underline{u} & \underline{dv} \\ \sin^{n-1} u & \sin u \end{array}$$

(1/2) EACH EXCEPT AS NOTED

$$(n-1) \sin^{n-2} u \cos u - \cos u$$

$$\begin{aligned} \int \sin^n u \, du &= \underline{-\sin^{n-1} u \cos u} + \underline{(n-1) \int \sin^{n-2} u \cos^2 u \, du} \\ &= -\sin^{n-1} u \cos u + (n-1) \int \sin^{n-2} u (1 - \sin^2 u) \, du \\ &= -\sin^{n-1} u \cos u + \underline{(n-1) \int \sin^{n-2} u \, du} - \underline{(n-1) \int \sin^n u \, du} \end{aligned}$$

$$\underline{n \int \sin^n u \, du = -\sin^{n-1} u \cos u + (n-1) \int \sin^{n-2} u \, du}$$

$$\underline{\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du} \quad (1)$$

Evaluate $\int x^2 \sqrt{x^2 - 4} dx$.

SCORE: ____ / 8 PTS

$$\underline{x = 2 \sec \theta \rightarrow \sec \theta = \frac{x}{2}}$$



$$dx = 2 \sec \theta \tan \theta d\theta$$

$$\int (2 \sec \theta)^2 \sqrt{4 \sec^2 \theta - 4} (2 \sec \theta \tan \theta) d\theta$$

$$= \underline{16 \int \sec^3 \theta \tan^2 \theta d\theta}$$

$$= 16 \int \sec^3 \theta (\sec^2 \theta - 1) d\theta$$

$$= \underline{16 \int \sec^5 \theta d\theta - 16 \int \sec^3 \theta d\theta}$$

$$= 16 \left(\frac{1}{4} \sec^3 \theta \tan \theta + \frac{3}{4} \int \sec^3 \theta d\theta \right) - 16 \int \sec^3 \theta d\theta$$

$$= 4 \sec^3 \theta \tan \theta - 4 \int \sec^3 \theta d\theta$$

$$= 4 \sec^3 \theta \tan \theta - 4 \left(\frac{1}{2} (\sec \theta \tan \theta + \ln |\sec \theta + \tan \theta|) \right) + C$$

$$= \underline{\frac{1}{2} 4 \sec^3 \theta \tan \theta - 2 \sec \theta \tan \theta - 2 \ln |\sec \theta + \tan \theta| + C}$$

$$= \underline{4 \left(\frac{x}{2} \right)^3 \frac{\sqrt{x^2 - 4}}{2} - 2 \left(\frac{x}{2} \right) \frac{\sqrt{x^2 - 4}}{2} - 2 \ln \left| \frac{x}{2} + \frac{\sqrt{x^2 - 4}}{2} \right| + C}$$

① EACH EXCEPT
AS NOTED

②

$$\frac{1}{4} x^3 \sqrt{x^2 - 4} - \frac{1}{2} x \sqrt{x^2 - 4} - 2 \ln \left| x + \sqrt{x^2 - 4} \right| + C$$